United Kingdom Mathematics Trust

British Mathematical Olympiad

Round 2 : Thursday, 31 January 2013

Time allowed *Three and a half hours.*

Each question is worth 10 marks.

Instructions

- *Full written solutions – not just answers – are required, with complete proofs of any assertions you may make. Marks awarded will depend on the clarity of your mathematical presentation. Work in rough first, and then draft your final version carefully before writing up your best attempt.*

  Rough work should *be handed in, but should be clearly marked.*
- *One or two* complete *solutions will gain far more credit than partial attempts at all four problems.*
- *The use of rulers and compasses is allowed, but calculators and protractors are forbidden.*
- *Staple all the pages neatly together in the top* left *hand corner, with questions 1, 2, 3, 4 in order, and the cover sheet at the front.*
- To accommodate candidates sitting in other timezones, please do not discuss any aspect of the paper on the internet until 8am GMT on Friday 1 February.

In early March, twenty students eligible to represent the UK at the International Mathematical Olympiad will be invited to attend the training session to be held at Trinity College, Cambridge (4–8 April 2013). At the training session, students sit a pair of IMO-style papers and eight students will be selected for further training and selection examinations. The UK Team of six for this summer's IMO (to be held in Santa Marta, Colombia, 18–28 July 2013) will then be chosen.

Do not turn over until **told to do so.**

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2012/13 British Mathematical Olympiad
Round 2

1. Are there infinitely many pairs of positive integers (m, n) such that both m divides $n^2 + 1$ and n divides $m^2 + 1$?

2. The point P lies inside triangle ABC so that $\angle ABP = \angle PCA$. The point Q is such that $PBQC$ is a parallelogram. Prove that $\angle QAB = \angle CAP$.

3. Consider the set of positive integers which, when written in binary, have exactly 2013 digits and more 0s than 1s. Let n be the number of such integers and let s be their sum. Prove that, when written in binary, $n + s$ has more 0s than 1s.

4. Suppose that $ABCD$ is a square and that P is a point which is on the circle inscribed in the square. Determine whether or not it is possible that PA, PB, PC, PD and AB are all integers.